

Making Plant-Like Fractals on GSP

by Mike M^cGarry
athenianmike@yahoo.com

Important Idea #1: Control Points

When you make a new line segment, circle, etc. on GSP, the points that determine it when it is created are “control points” — they control the size & position of that object.

Important Idea #2: Minimize Control Points

When you making any fractal on GSP, you want as few independent control points as possible. Often, you want only two, no more. You will construct the “stage zero” line segment, with its two control points, and then all other points will be determined by those two points (via midpoints, dilations, rotations, etc.) Once you use the “line segment” tool to build your first line segment, you don’t want to touch that tool again.

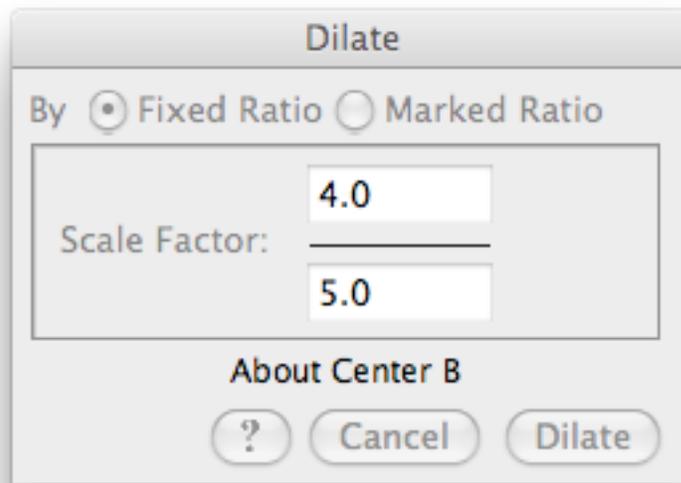
Important Idea #3: Clicking Off

If you are new to GSP, after each action, I suggest getting in the habit of clicking the mouse on a blank area of the screen. That will de-select everything. You see, whenever you create a new point or line, it pops into existence as already highlighted, already selected. If you jump immediately into your next move, you are going to be involving the point or line you just created, simply because it’s still highlighted. You will wind up constructing something you don’t want to construct, or hiding something you don’t want to hide. If you routinely click off, de-selecting everything, you never have any of those accidents.



A step-by-step guide

- 1) In a new GSP document, make a vertical line segment. Call the top A and the bottom B. A & B are going to be the only control points in this entire diagram.
- 2) Construct the midpoint (highlight segment, ⌘-M)
- 3) Double-click on point B. You will see the momentary “radioactive” glow around it, indicating it has been marked as a center. Alternately, you could simply highlight B, and under the Transform menu, choose “Mark Center.”
- 4) Select the midpoint, and under the Transform menu, choose dilate. The “Dilate” box will appear:



Choose some dilation factor slightly less than 1. Here, I chose $4/5$. Then press dilate.

- 5) Without clicking off, immediately go back to the Transform menu, Dilate, and hit return. Do this repeatedly, so you dilate each successive point and wind up with a line of points. (You can do this quickly by using the right hand to select the option Dilate with the mouse, and simply keeping the left hand at the “return” key to hit enter every time the Dilate box comes up.)



6) Now, I am going to be efficient/lazy (depending on your perspective). Double-click on the midpoint, to mark it as a center. Then select all those newly created points between the midpoint and B. Go to Transform, Rotate, and rotate by 180° . That will put them all above the midpoint, between the midpoint and A.

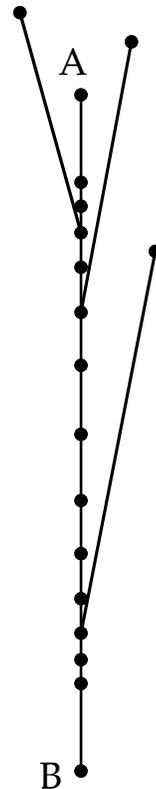
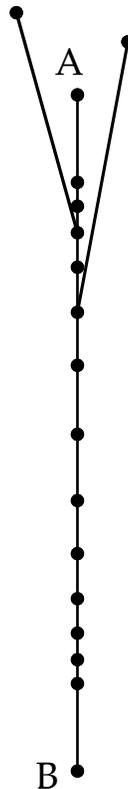
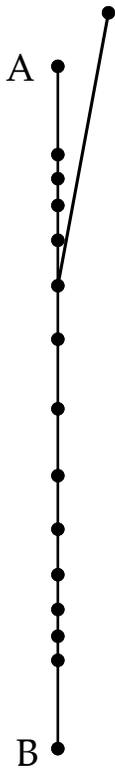
7) The whole point of steps #2-7 was to create a variety of points between A and B that are wholly dependent on A & B — i.e., no new control points created. You could also do this with midpoints of smaller and smaller segments, or in a number of other ways.

8) Now that I have all these points on the trunk, I am going to start process — in general, the process is: pick one of those points, any that strikes my fancy, mark it as center, then pick another one of these points, rotate it by a certain angle, and connect the image to a point (either the first picked or a third new point) already on the “trunk” to create a “branch.” Steps #9-13 will demonstrate this process. The points & angles you pick will determine the kind of shape that results: with practice, you will get a sense of what initial shapes lead to what final products.

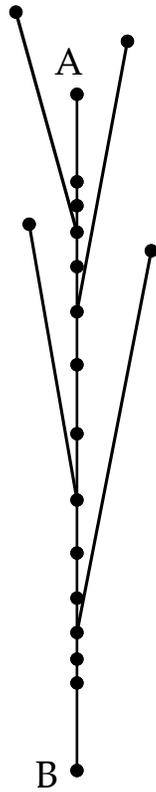
9) For example: I marked the third point below A as a center. Then I selected the midpoint of \overline{AB} , went to Transform-Rotate, and rotated by 165° . Then I connect this to the fifth point below A.

10) Then, I marked the point immediately below A as the center, selected the point just above the midpoint, and rotated it by -160° (negative, so it would be on the other side). Connected it to third point below A.

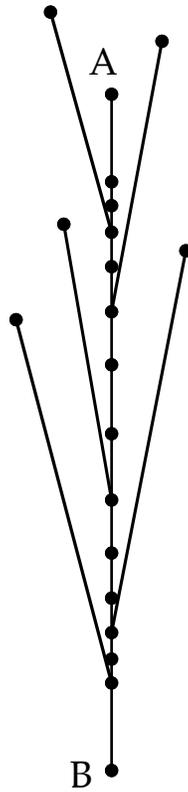
11) Then, I marked A as the center, select the fourth point below A, and rotated by 25° . Then connected it to the third point above B.



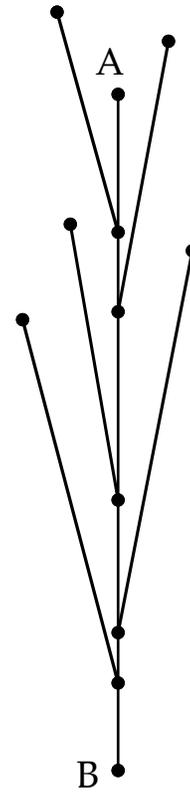
12) Still with A as the center, I selected the third point below A, rotated by -20° , and connected it to the point below the midpoint.



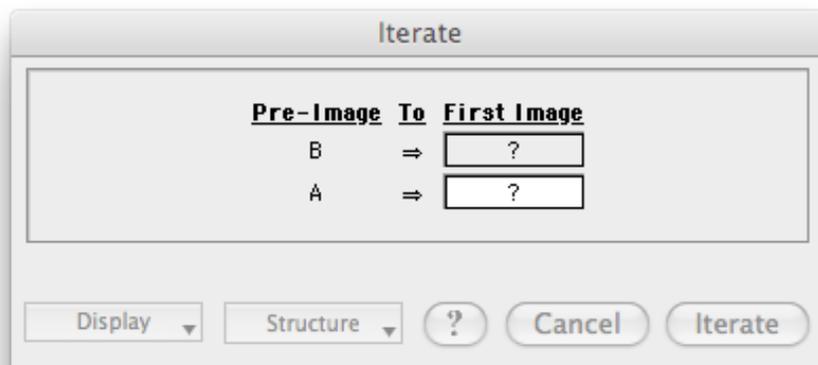
13) Mark B as the center. Select the point above the midpoint. Rotate by 12° . Connect to the point directly above B.



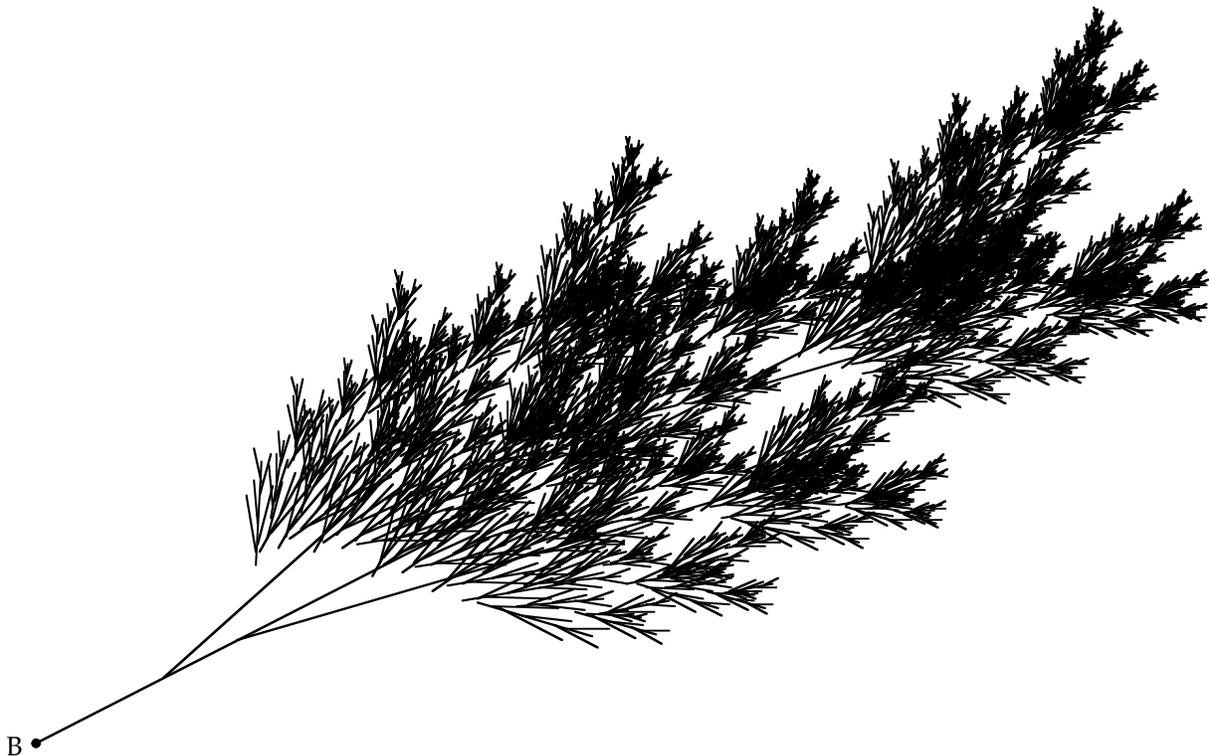
14) Hide every point on the trunk that is not at the base of a branch. Make sure, when you are done, every branch still has two endpoints, and that A & B are still there.



15) Now, very important: after clicking off, select B, then A. Then go to the Transform menu, and choose the bottom option: **Iterate**. The iteration box will appear.



- 16) Simply click on the base of any branch, then on the tip of that branch. Names of points will appear in the boxes to the right of A and B in the Iterate box.
- 17) Then, go to the “Structure” tab, and select “Non-Point Images Only.” You will see points at the tips of the little phantom branches disappear.
- 18) Then, go the “Display” tab, and select “Final Iteration Only.” The welter of phantom branches will thin out, becoming more delicate.
- 19) Then (a) hit -A to add a new “map” in the Iteration box, (b) click on the base of another branch, and then (c) click on the tip of that branch. Repeat this for each branch, treating the tiny little vertical stretch between A and the next point down as a branch.
- 20) Hit the plus key (or under “Display” tab, select “Increase Iterations”). Do this, until you get the message that there’s not enough memory for more iterations. Then, hit the “Iterate” button.
- 21) I like to select the whole thing, and under the Display menu, go to Color, and change the color to green. This also makes it easier to see all the points, the bases & tips of branches. Hide these points.



Again, from here, experiment on your own branches, using other angles and other lengths. With practice, you will develop intuition for these shapes.

Appendix

A particularly non-rigorous comment about the finding the dimensions of fractals.

First, let's review a little about dimension.

- If you have a line segment (1D), and triple the length, you have 3 times as much line segment.
- If you have a square (2D), and triple each length, you have 9 times as much square.
- If you have a cube (3D), and triple each length, you have 27 times as much cube.
- Presumably, if you have a hypercube (4D), and triple each length, you have $3^4 = 81$ times as much hypercube.

Generalizing from 3 to scale factor k

- If you have a line segment (1D), and multiply the length by k, you have k times as much line segment.
- If you have a square (2D), and multiply each length by k, you have k^2 times as much square.
- If you have a cube (3D), and multiply each length by k, you have k^3 times as much cube.
- Presumably, if you have a hypercube (4D), and multiply each length by k, you have k^4 times as much hypercube.

Generalizing even more

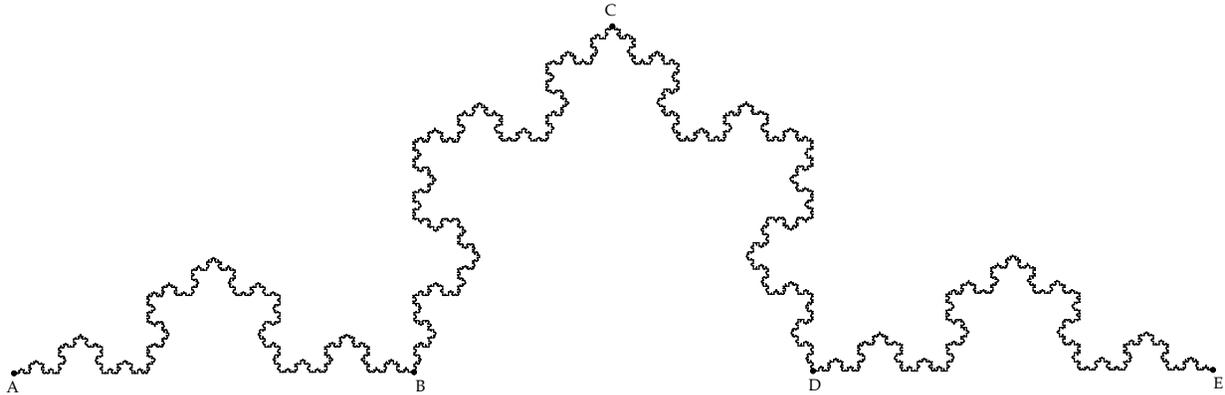
- If you have a d-dimensional object, and multiply the length by k, you have k^d times as much of that thing.

In equation form,

$$k^d = S$$

Where k = scale factor, d = dimension, and S = how many times amount of stuff has increased. This equation, in essence, is a practical definition for dimension.

Now, let's apply that to a fractal.



This is a section of a famous fractal, the Koch Snowflake.

The section from A to B is a miniature copy of the whole thing: an instance of self-similarity across scale.

Let's consider the scaling operation, as if we started with the part from A to B, and scaled it up to get the whole picture.

The distance AB is 1/3 of the distance AE, so the scaling factor is $k = 3$.

When we scale up, starting with the part from A to B, and ending with the whole picture, how much more "stuff" do we have? Well, think of it this way: suppose you had a number of stickers with a picture of the part from A to B. How many of those stickers would it take to build the entire picture above? Four. You would need one stick to make the part from A to B, one for the part from B to C, one for the part from C to D, and a final one for the part from D to E. So, $S = 4$.

$$k^d = S$$

$$3^d = 4$$

$$d \log(3) = \log(4)$$

$$d = \frac{\log(4)}{\log(3)} = 1.261859507 \dots$$

This is called the Hausdorff dimension. The Koch Snowflake has a Hausdorff dimension of 1.261859507 . . .